Shape phase transitions and critical points in odd nuclei

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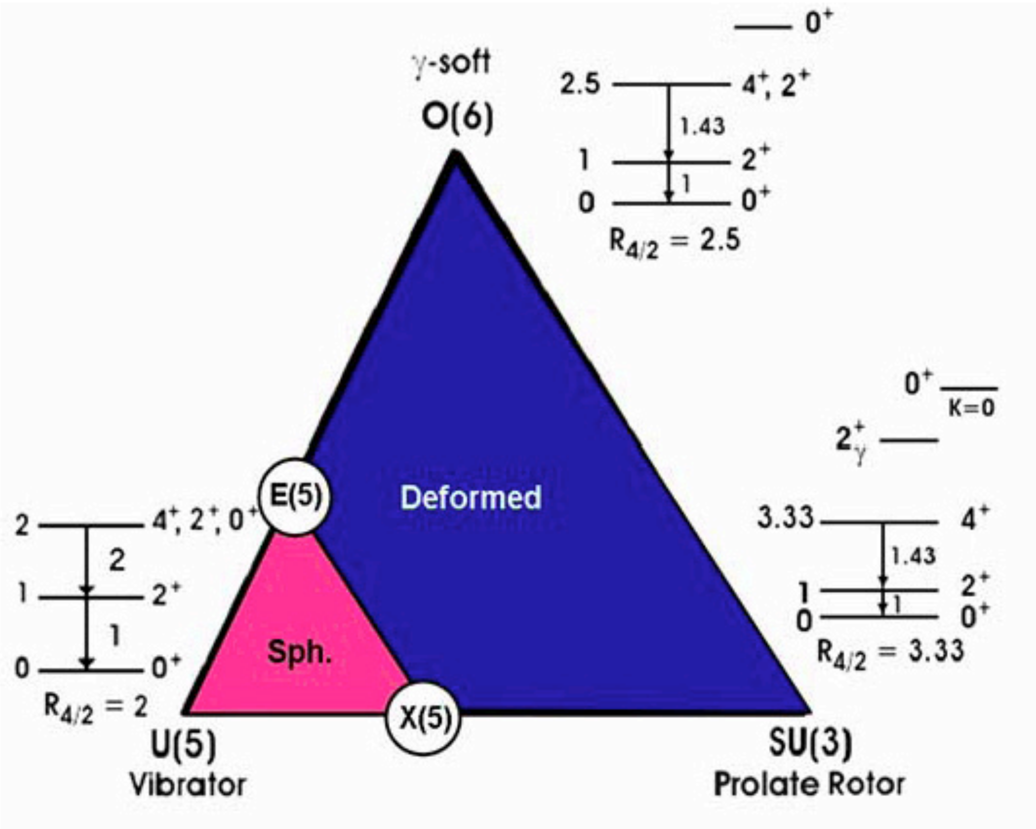
Different regions of the nuclear chart can be usefully characterized by resorting to the macroscopic concept of nuclear shape: we are therefore used to speak of spherical nuclei, axially deformed, triaxial nuclei, gamma-unstable, octupole, etc

Nuclei are of course much more complicated than that. A full microscopic description of the nuclear many-body system is often needed to describe the full variety of nuclear behaviour.

Yet, simple models can describe the main gross features along the nuclear chart. One of such simple models is the Interacting Boson Model (IBM).

By varying the parameters of the IBM hamiltonian one can cover the main features of the quadrupole degree of freedom. This is pictorially shown in the so-called Casten triangle, where each point represents a definite physical situation and the vertexes correspond to the three analitycal solutions of the IBM.

Shape diagram



By varying the number of neutrons or protons along isotope or isotone chains one can move from one shape to another.

Interest has been recently posed (first theoretically and later experimentally) on the possibility of sharp shape transitions and on the nuclear behaviour at the critical points.

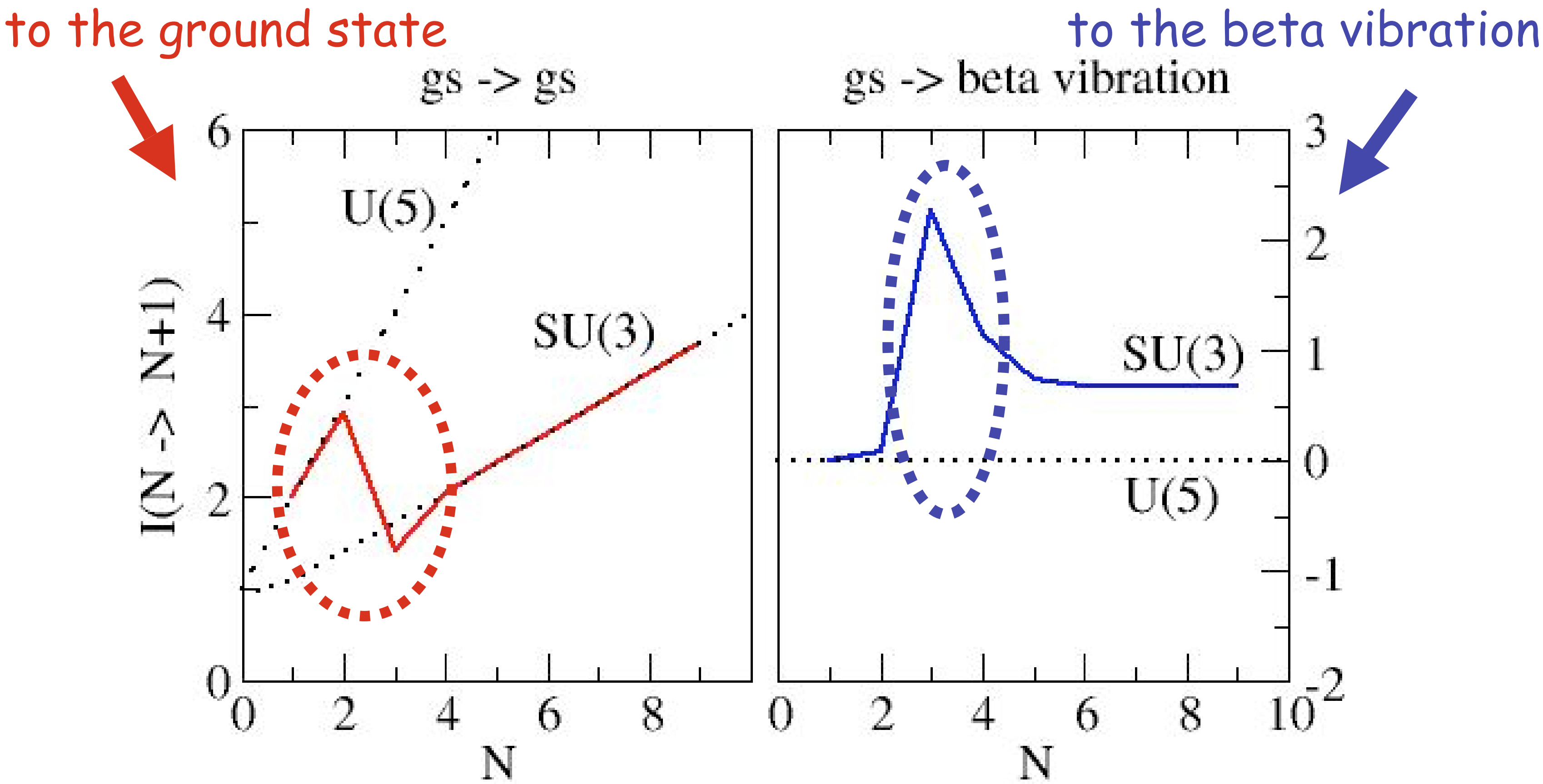
Possible special analytic solutions have been advanced at the critical points (so-called critical point symmetries)

Signatures of the phase transition in even-even nuclei:

- Spectrum (not only the usual ratio E_4/E_2 in the ground band, but in particular the excitation energy of the β-band)
- $B(E\lambda)$'s (in-band and intra-band)
- Two-particle transfer matrix elements

Example: L=0 pair transfer in a phase transition from spherical to axial deformation

(from Fossion etal, PRC, 2007)



What about the neighbour odd nuclei?

Few questions:
Their behaviour will reflect the phase transition in the even nuclei? In other words, there will be a critical point also in the odd nuclei?

In case of positive answer, will the position and the properties of the (even) critical point be affected by the coupling to the extra particle?

The U(5) to O(6) transition

(from sphericity to deformation
with gamma-instability)

Within the IBM this transition can be obtained in even-even nuclei for example from the hamiltonian

$$
\begin{aligned}
H^B &= x\hat{n}_d - \frac{1-x}{N}\hat{Q}_B \times \hat{Q}_B \\
&= xC_1(U^B(5)) - \frac{1-x}{2N}\left[C_2(O^B(6) - C_2(O^B(5)\right]
\end{aligned}
$$

$$
\hat{n}_d = \sum_\mu d^\dagger_\mu d_\mu \quad ; \quad \hat{Q}_B = \left(s^\dagger \tilde{d} + d^\dagger \tilde{s}\right)^{(2)}
$$

Varying the control parameter x we get a transition from spherical to deformed gamma-unstable shape: but where does the transition take place?

To answer this question one has to resort to the concept of intrinsic states and potential energy surfaces

$$E_N(\beta,\gamma) = \langle g;N,\beta,\gamma|\hat{H}|g;N,\beta,\gamma\rangle$$

within the condensate $$|g;N,\beta,\gamma\rangle = \frac{1}{\sqrt{N!}}(\Gamma_g^\dagger)^N|0\rangle$$

where $$\Gamma_g^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_2^\dagger + d_{-2}^\dagger)\right]$$

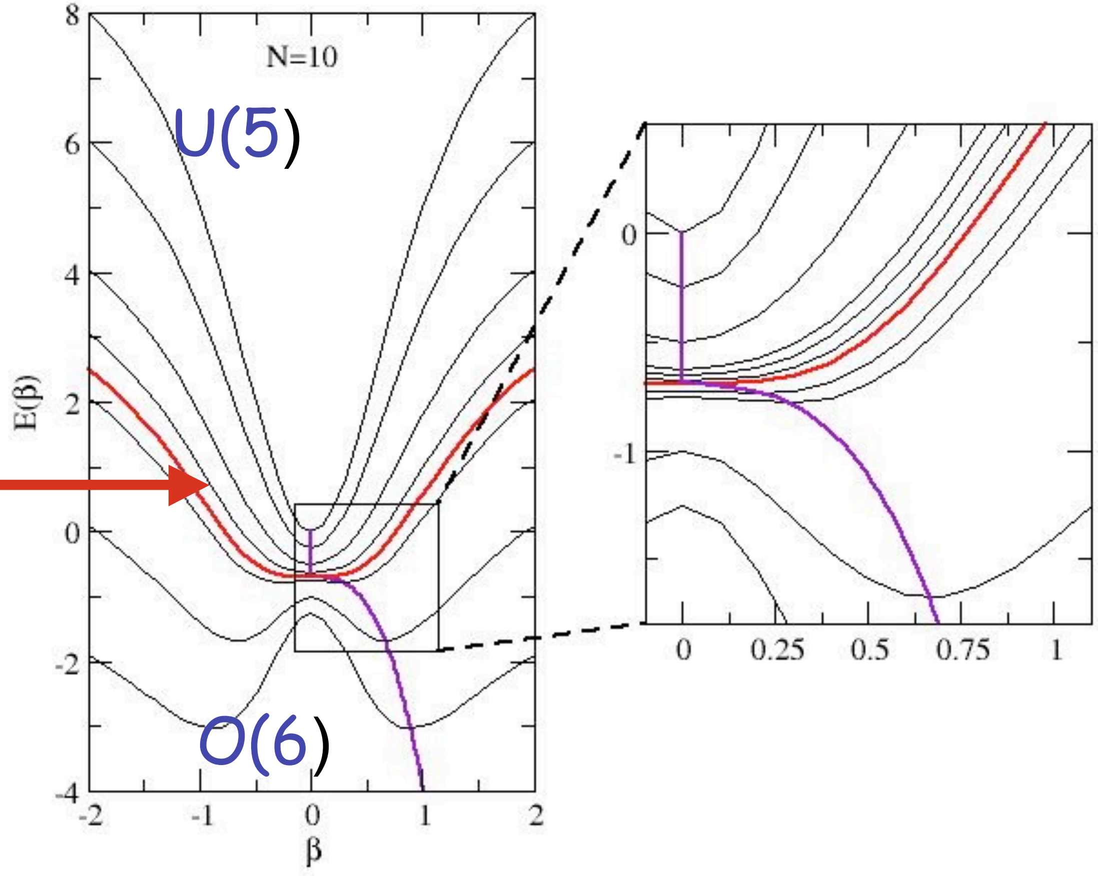
Energy surfaces E(β)

γ-independent for any value of x

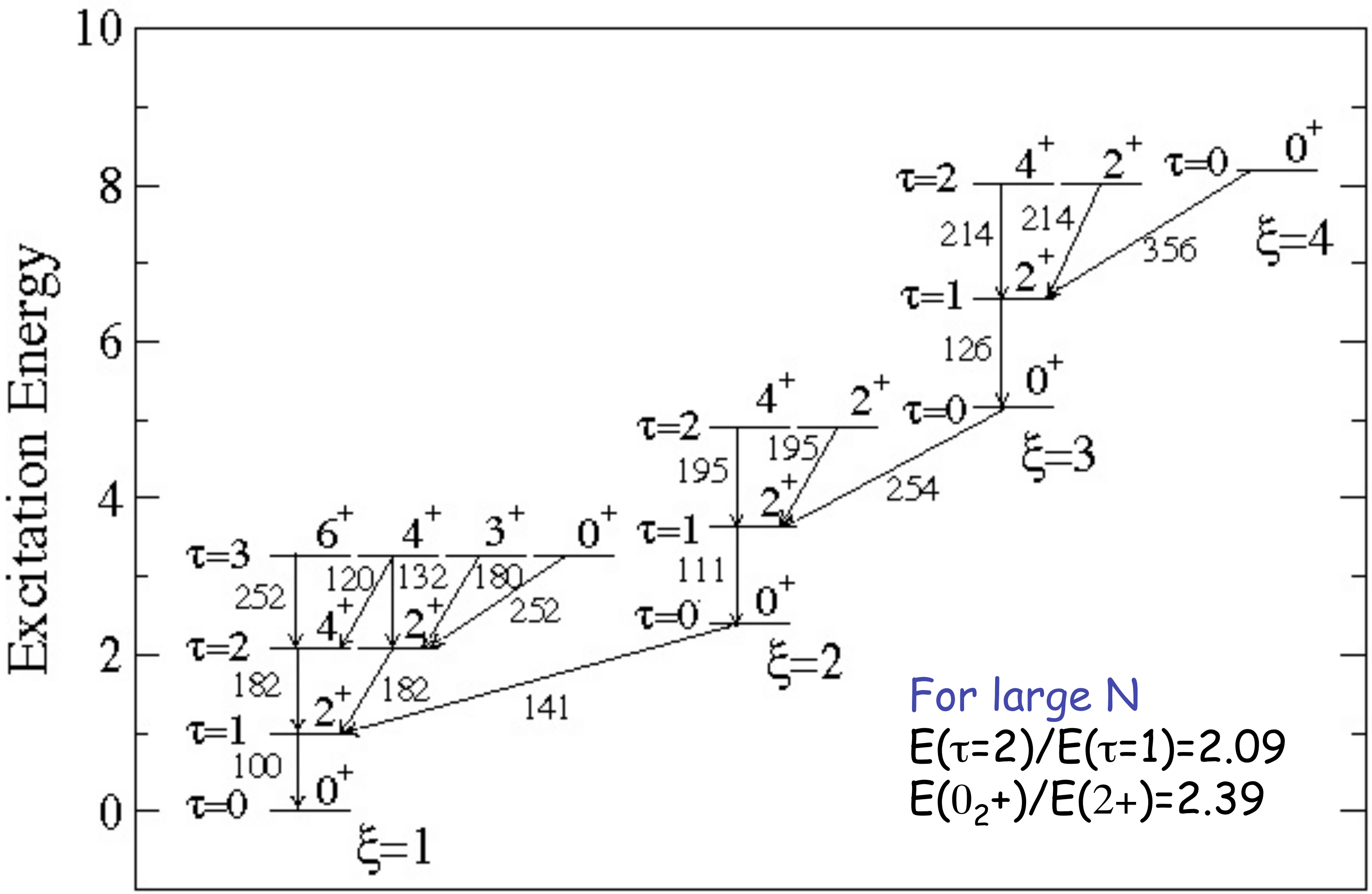
U(5) to O(6) transition
(varying the value of x)

critical point

(second-order phase transition)



Spectrum at the critical point



One can address the same physics within the Bohr hamiltonian

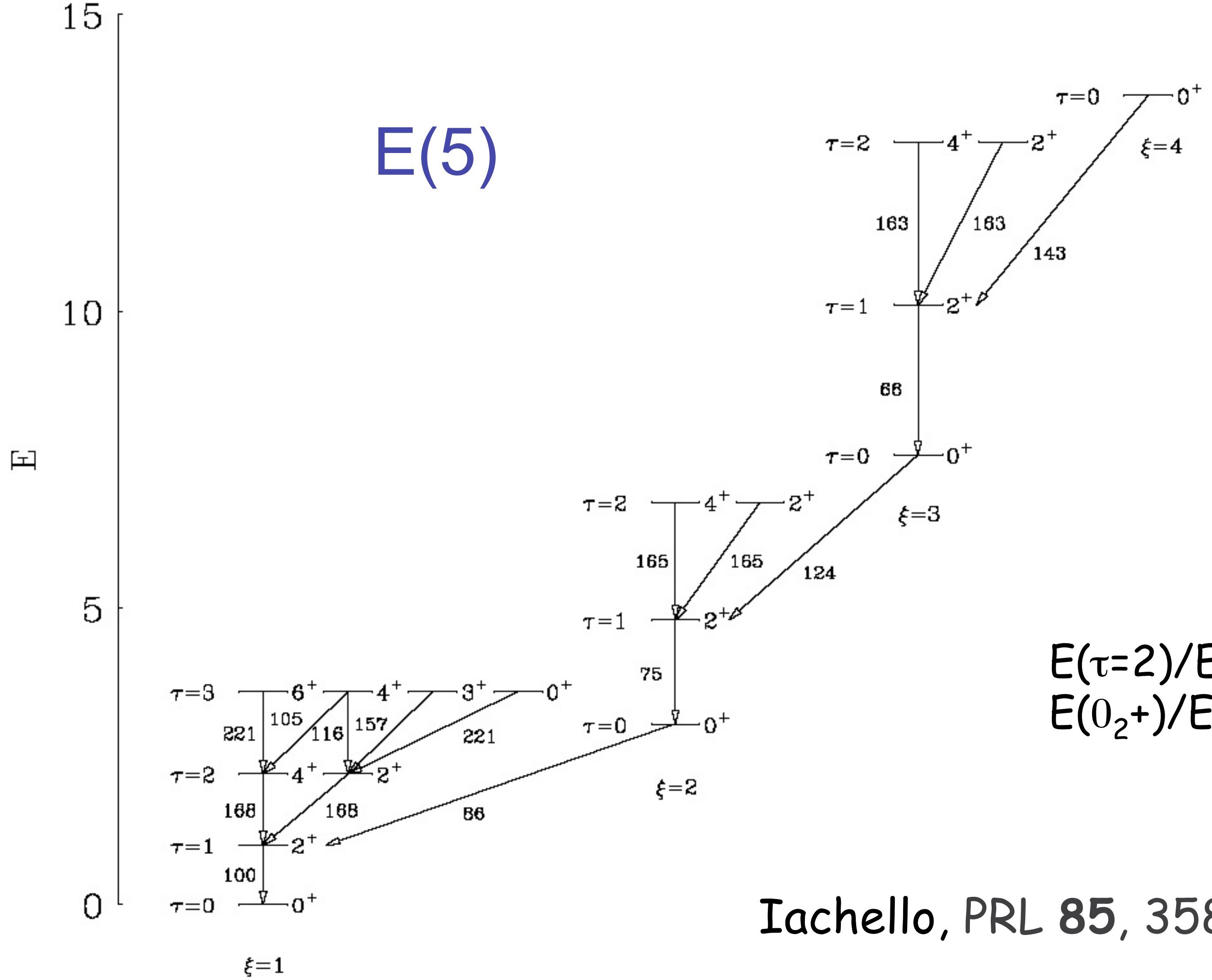
$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^4}\sum_\kappa \frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + u(\beta)$$

Using the information of a rather flat (in β) and γ-independent potential and assuming the extreme case of an infinite square well

$$u(\beta) = 0, \qquad \beta < \beta_w;$$
$$u(\beta) = \infty, \qquad \beta \geq \beta_w$$

one reaches the so-called E(5) situation

E(5)



E(τ=2)/E(τ=1)=2.20
E(0_2+)/E(2+)=3.03

Iachello, PRL **85**, 3580 (2000)

What happens when we couple the even-even core to an odd particle? In general this coupling destroys the situation of γ-instability. With some noticeable exceptions, that preserve the γ-instability along the whole shape transition (including the critical point):

j=3/2

Iachello, PRL **95**, 052503 (2005) E(5/4)

Alonso,Arias,Fortunato,Vitturi, PRC **72**, 061302 (2005)

j=1/2,3/2,5/2

$(j=\tilde{l}+\tilde{s},\ \tilde{l}=0,2,\ \tilde{s}=1/2)$

Alonso,Arias,Vitturi, PRL **98**, 052501 (2007) E(5/12)

Alonso,Arias,Vitturi, PRC **75**, 064316 (2007)

Following the previous lines, the odd system is described within the Interacting Boson Fermion Model (IBFM)

$$H = H_B + H_F + V_{BF}$$

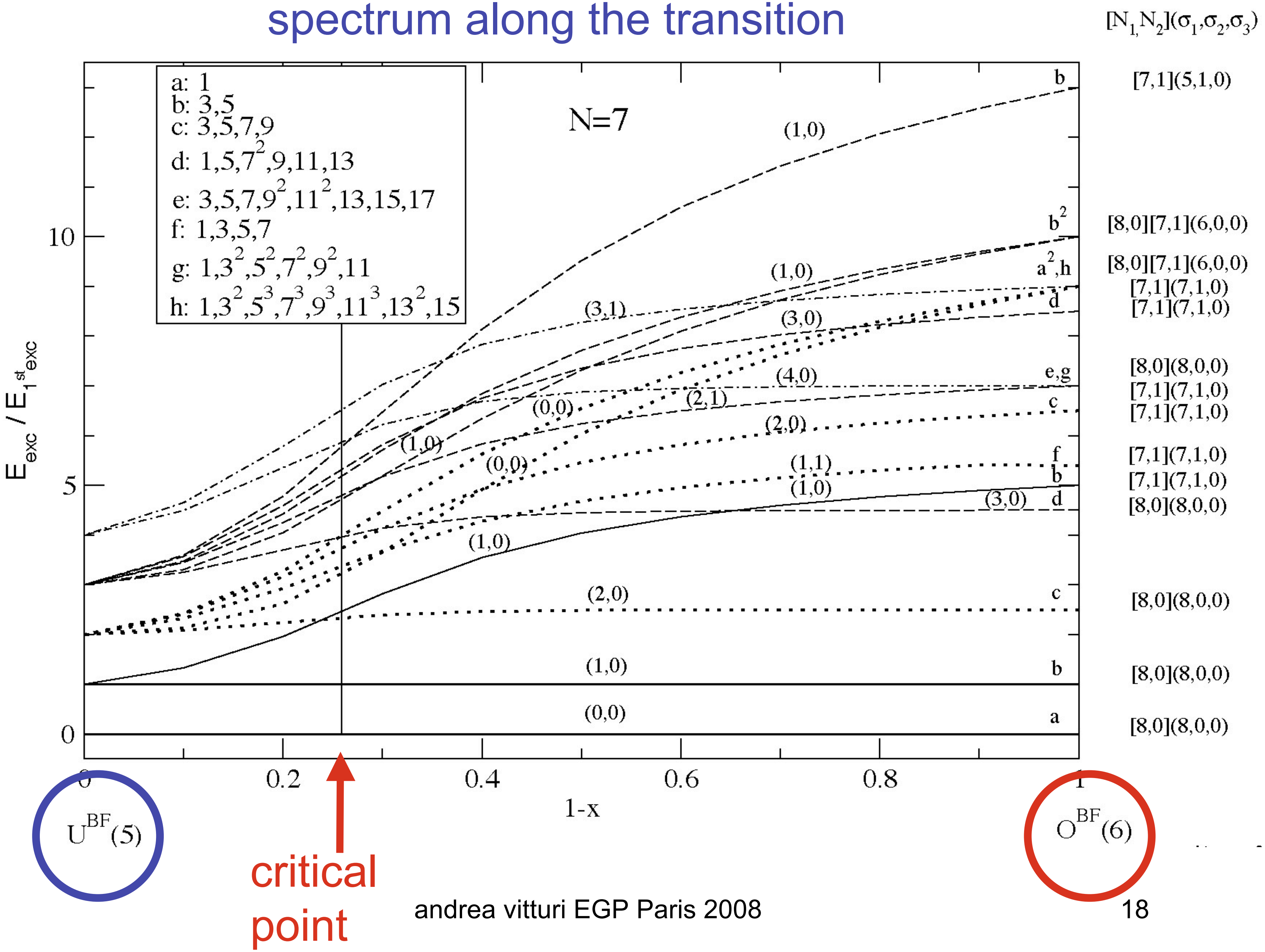
adding to the previous boson hamiltonian the terms

$$H_F + V_{BF} = \sum_j \epsilon_j a_j^\dagger \cdot \tilde{a}_j - 2\frac{1-x}{N}\hat{Q}_B \cdot \hat{q}_F.$$

With proper choices of the single-particle energies and fermion quadrupole operator the total Boson-Fermion hamiltonian can be recast in the form

$$H_{BF} = xC_1(\mathrm{U}^{BF}(5)) - \frac{1-x}{2N}[C_2(\mathrm{O}^{BF}(6)) - C_2(\mathrm{O}^{BF}(5))].$$

spectrum along the transition



As in the case of even nuclei a better understanding can be obtained by resorting to the intrinsic states and the corresponding energy surfaces.

Intrinsic states can be obtained by diagonalizing the boson-fermion interaction in the basis states obatined by coupling the odd particle to the even intrinsic states (depending on β and γ)

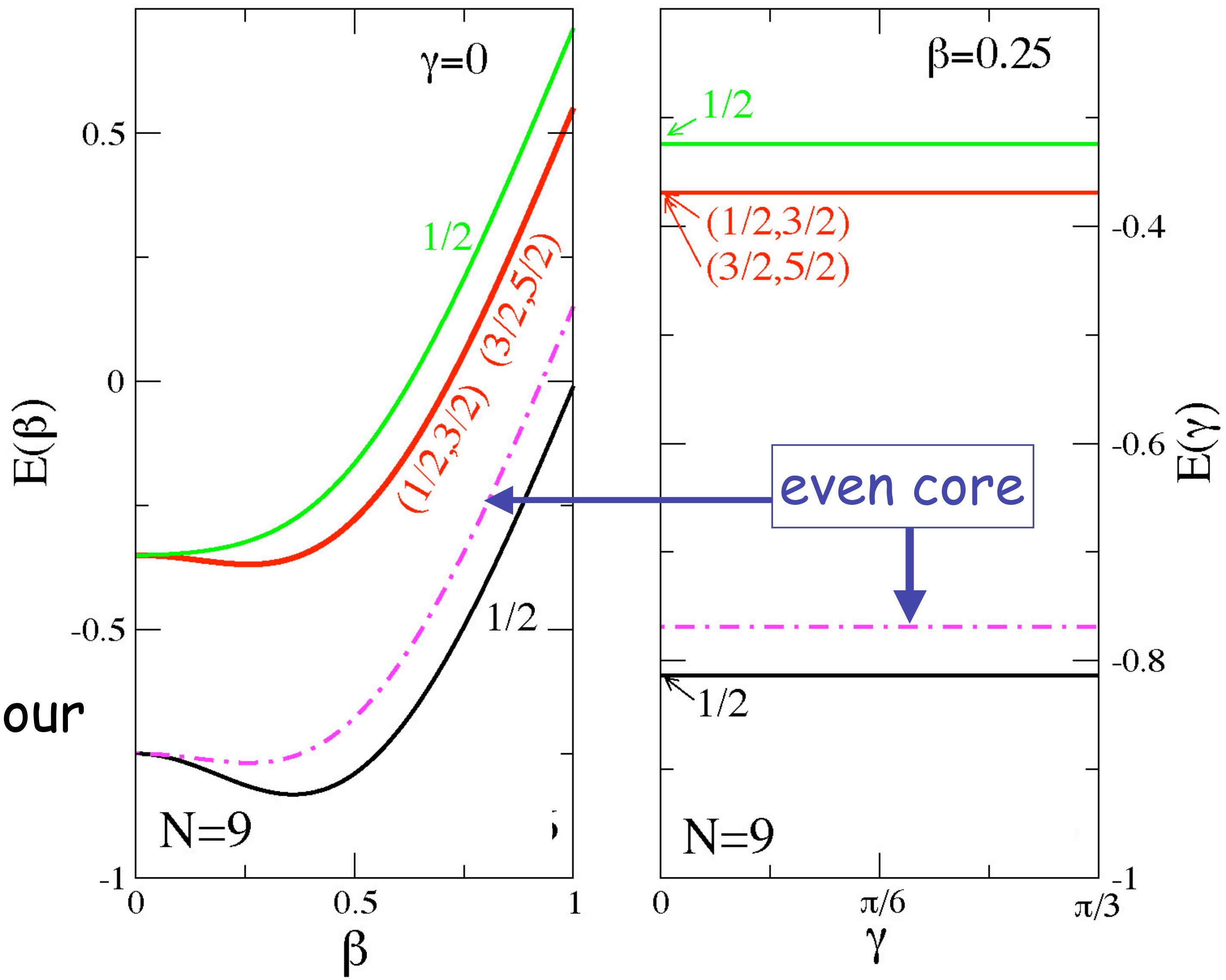
$$\Psi_n(\beta,\gamma) = \Phi_{gs}(\beta,\gamma) \otimes |jk>$$

As expected the energy surfaces are γ-independent and, at the critical point, rather flat in the β variable.

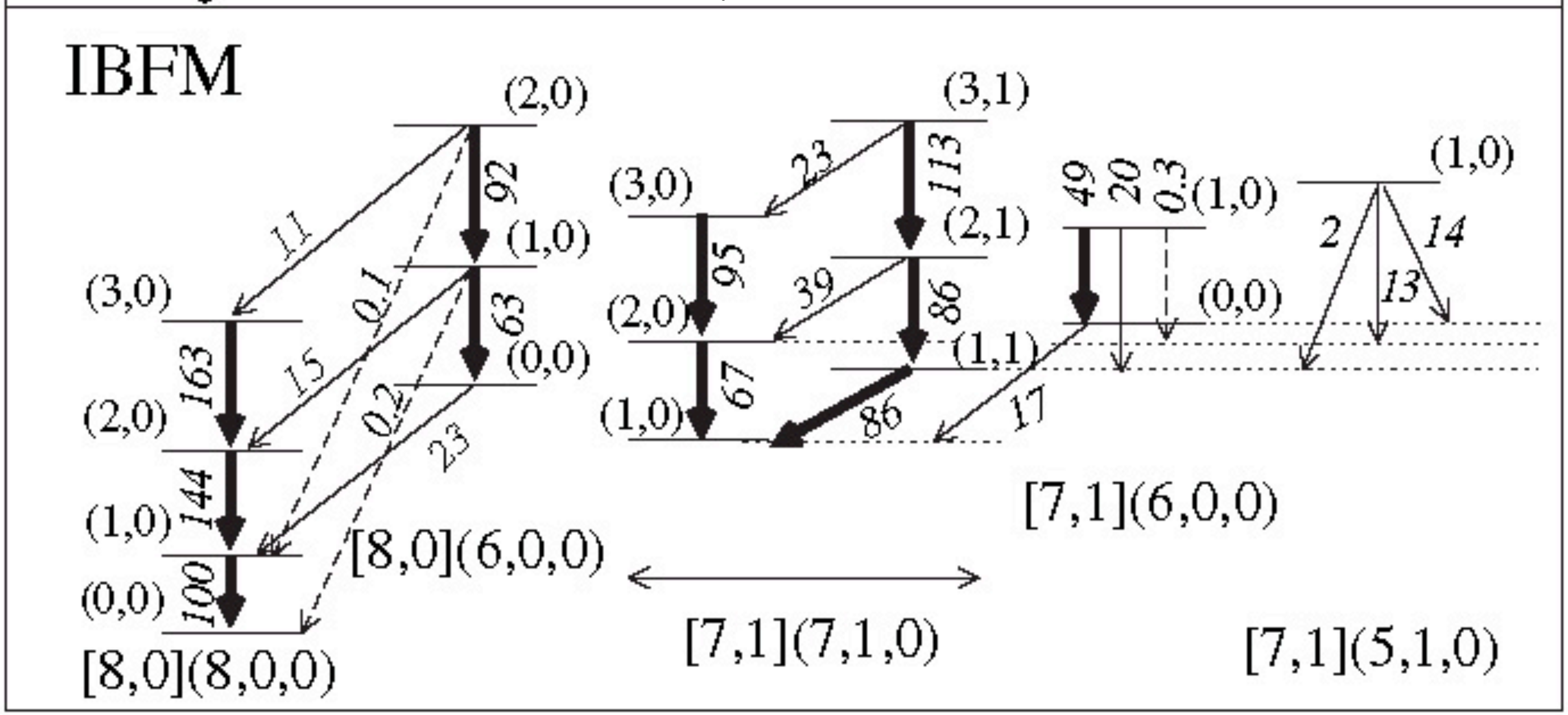
Each intrinsic state gives rise, in the laboratory, to a "triangular" band (a la O(5))

At the critical point

flat
behaviour



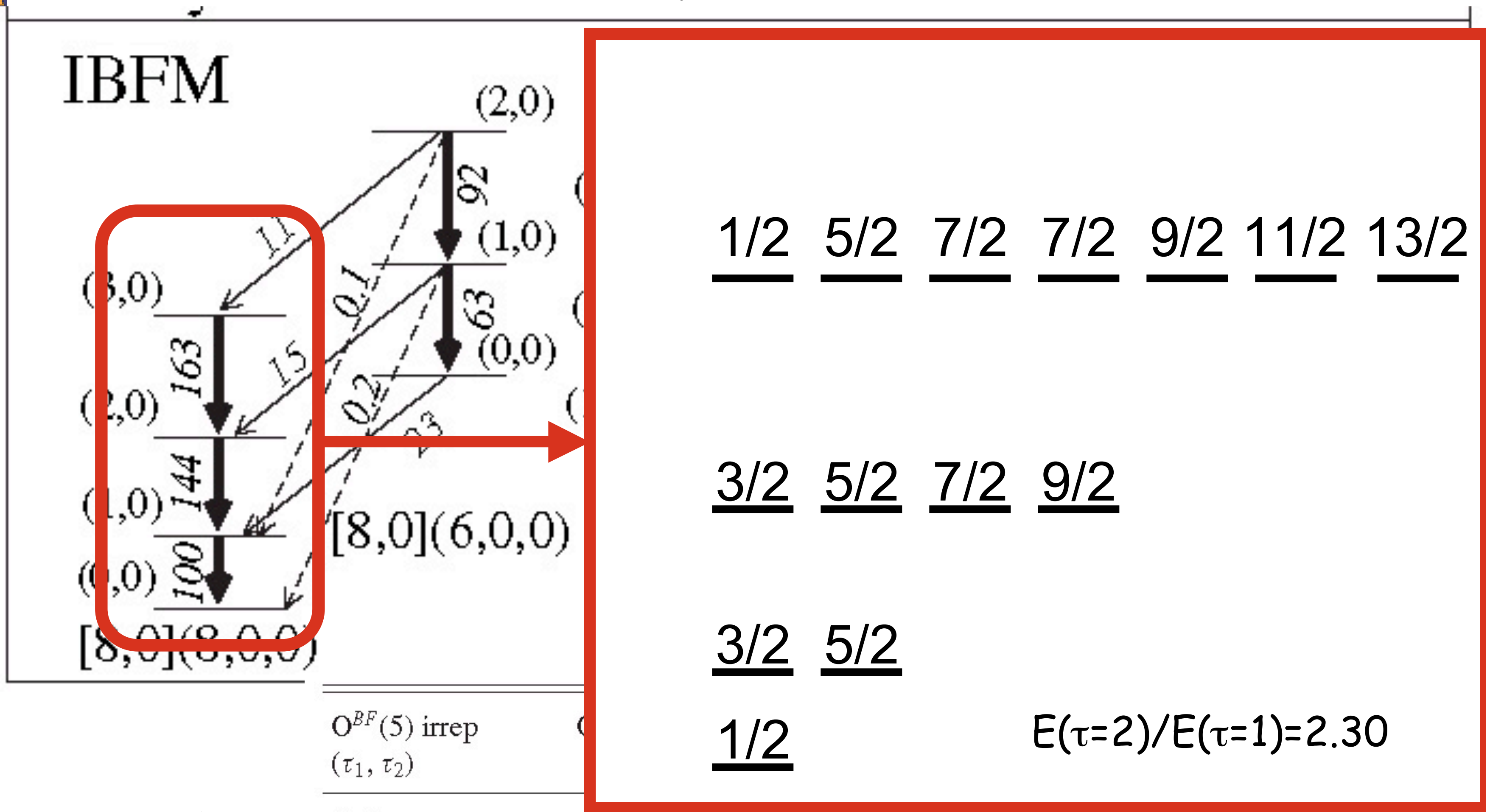
At the critical point x≈0.74 (N=7)



Angular momenta in degenerate levels

$O^{BF}(5)$ irrep (τ_1, τ_2)	$O^{BF}(3)$ irrep L_{BF}	Total ang. mom. J
(0,0)	0	1/2
(1,0)	2	5/2,3/2
(2,0)	4,2	9/2,7/2,5/2,3/2
(3,0)	6,4,3,0	13/2,11/2,9/2,$(7/2)^2$,5/2,1/2
(4,0)	8,6,5,4,2	17/2,15/2,13/2,$(11/2)^2$,$(9/2)^2$,7/2,5/2,3/2
(1,1)	3,1	7/2,5/2,3/2,1/2
(2,1)	5,4,3,2,1	11/2,$(9/2)^2$,$(7/2)^2$,$(5/2)^2$,$(3/2)^2$,1/2
(3,1)	7,6,5^2,4,3^2,2,1	15/2,$(13/2)^2$,$(11/2)^3$,$(9/2)^3$,$(7/2)^3$,$(5/2)^3$,$(3/2)^2$,1/2

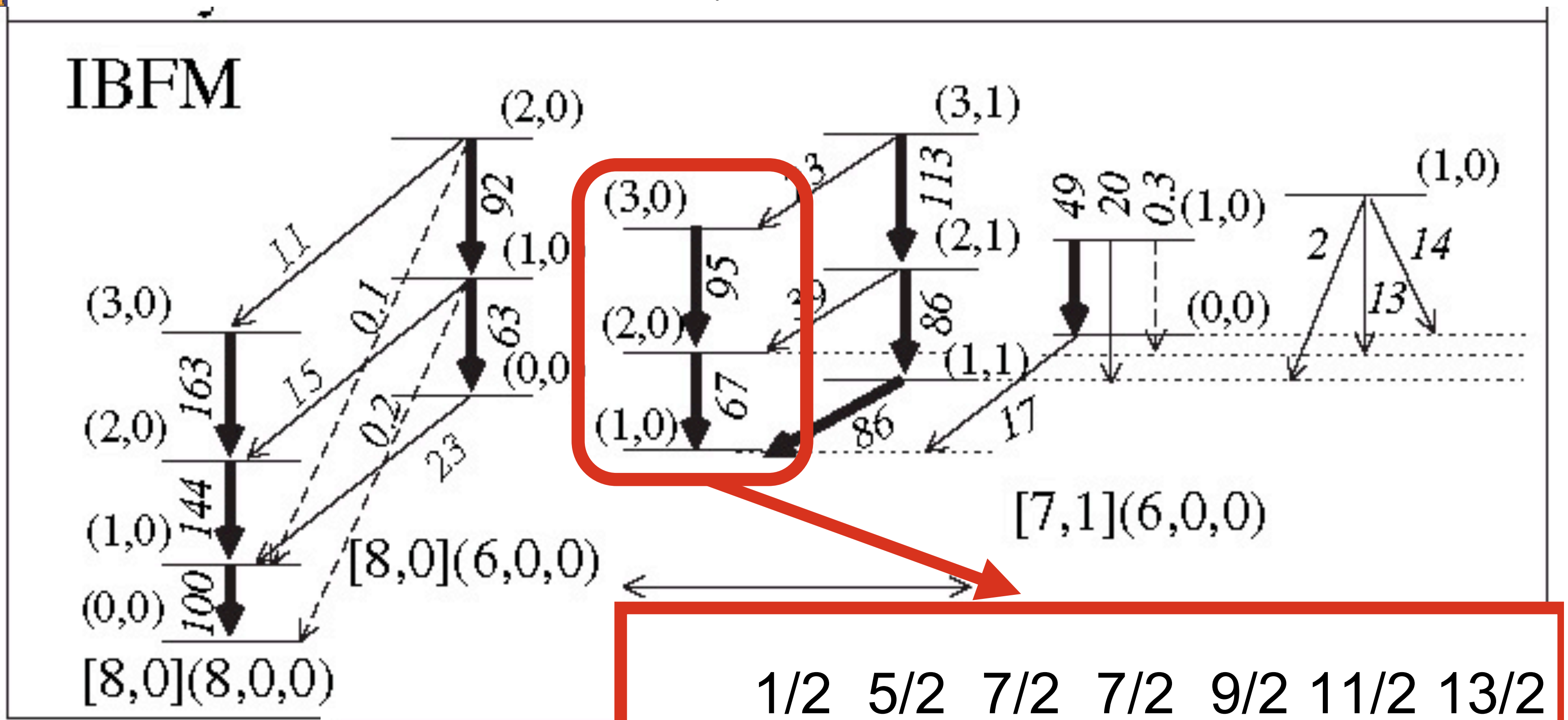
At the critical point x≈0.74 (N=7)



$O^{BF}(5)$ irrep (τ_1, τ_2)		
(0,0)	0	1/2
(1,0)	2	5/2,3/2
(2,0)	4,2	9/2,7/2,5/2,3/2
(3,0)	6,4,3,0	13/2,11/2,9/2,$(7/2)^2$,5/2,1/2
(4,0)	8,6,5,4,2	17/2,15/2,13/2,$(11/2)^2$,$(9/2)^2$,7/2,5/2,3/2
(1,1)	3,1	7/2,5/2,3/2,1/2
(2,1)	5,4,3,2,1	11/2,$(9/2)^2$,$(7/2)^2$,$(5/2)^2$,$(3/2)^2$,1/2
(3,1)	7,6,5^2,4,3^2,2,1	15/2,$(13/2)^2$,$(11/2)^3$,$(9/2)^3$,$(7/2)^3$,$(5/2)^3$,$(3/2)^2$,1/2

Angular momenta in degenerate levels

At the critical point x≈0.74 (N=7)



$O^{BF}(5)$ irrep (τ_1, τ_2)
(0,0)
(1,0)
(2,0)
(3,0)
(4,0)
(1,1)
(2,1)
(3,1) 7,

Angular momenta in degenerate levels

1/2 5/2 7/2 7/2 9/2 11/2 13/2

3/2 5/2 7/2 9/2

3/2 5/2

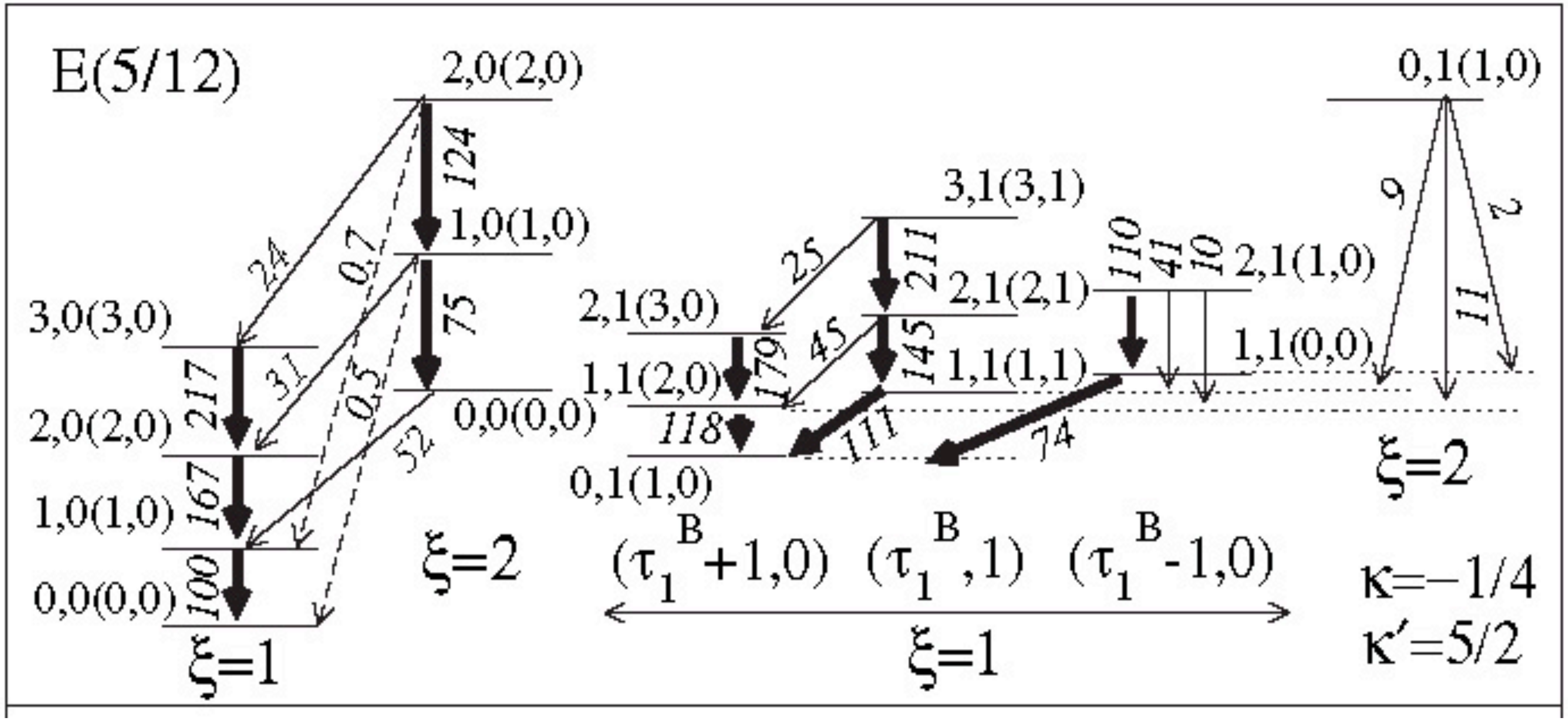
The flat behaviour of the energy surfaces suggests the idea of an approach similar to that used for the E(5) case. One introduces a collective hamiltonian which couples the even core to the odd particle

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^4}\sum_\kappa \frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + u(\beta) + kg(\beta) \times [2\hat{\mathcal{L}}_B \circ \hat{\mathcal{L}}_F] + k'g(\beta)\hat{\mathcal{L}}_F^2,$$

boson-fermion part

Using an infinite square well for u(β) the hamiltonian is analitycally solvable (E(5/12))

Alonso,Arias,Vitturi, PRL **98**, 052501 (2007)

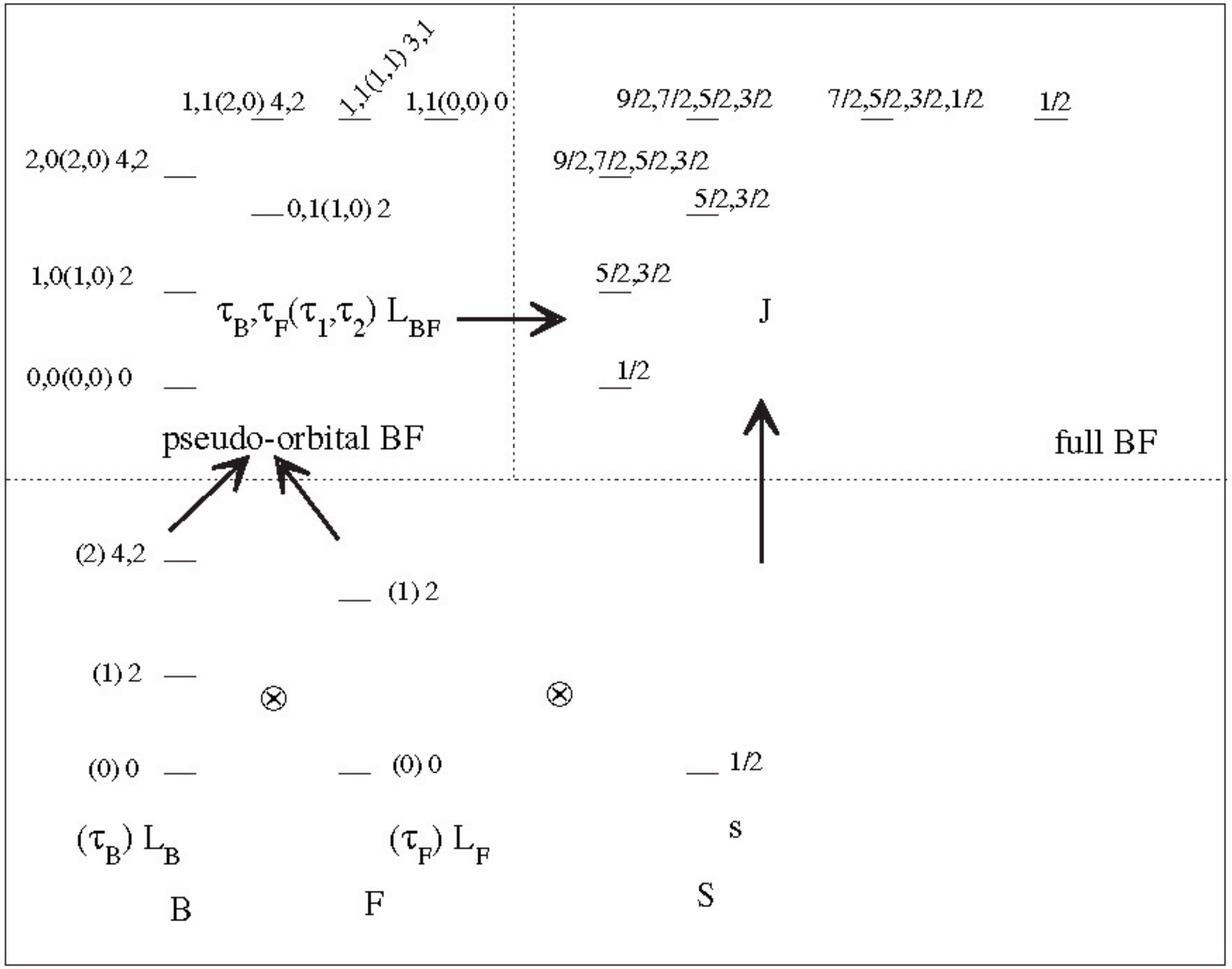


E(τ=2)/E(τ=1)=2.20

Obs: spectrum and B(Eλ) qualitatively similar to those obtained at the critical point within the IBFM

Coupling scheme

(boson+fermion pseudo-orbital+fermion pseudo-spin)



Possible candidates for E(5/12) critical point symmetry

Nucleus	Single particle orbitals
Odd-neutron Pt	$3p_{1/2}, 3p_{3/2}, 2f_{5/2}$
Odd-proton Ir	$3s_{1/2}, 2d_{3/2}, 2d_{5/2}$
Odd-neutron Ba	$3s_{1/2}, 2d_{3/2}, 2d_{5/2}$

The U(5) to SU(3) transition

(from sphericity to axial deformation)

Within the IBM this transition can be obtained in even-even nuclei for example from the hamiltonian

$$
\begin{aligned}
H^B &= (1-x)n_d - \frac{x}{4N_B} Q_B.Q_B \\
&= (1-x)\ C_1(U^B 5) \\
&- \frac{x}{8N_B}\left[\frac{3}{2}\ C_2(SU^B 3) - \frac{3}{8}\ C_2(O^B 3)\right]
\end{aligned}
$$

with the boson quadrupole operator

$$
Q_B = (s^\dagger \times \tilde{d})^{(2)} + (d^\dagger \times \tilde{s})^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})^{(2)}
$$

As in the previous case the value of the control parameter x where the transition from spherical to stable deformed shape takes place can be obtained by resorting to the concept of intrinsic states and potential energy surfaces

$$E_N(\beta, \gamma) = \langle g; N, \beta, \gamma | \hat{H} | g; N, \beta, \gamma \rangle$$

within the condensate $$|g; N, \beta, \gamma\rangle = \frac{1}{\sqrt{N!}} (\Gamma_g^\dagger)^N |0\rangle$$

where $$\Gamma_g^\dagger = \frac{1}{\sqrt{1+\beta^2}} \left[s^\dagger + \beta \cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}} \beta \sin\gamma (d_2^\dagger + d_{-2}^\dagger) \right]$$

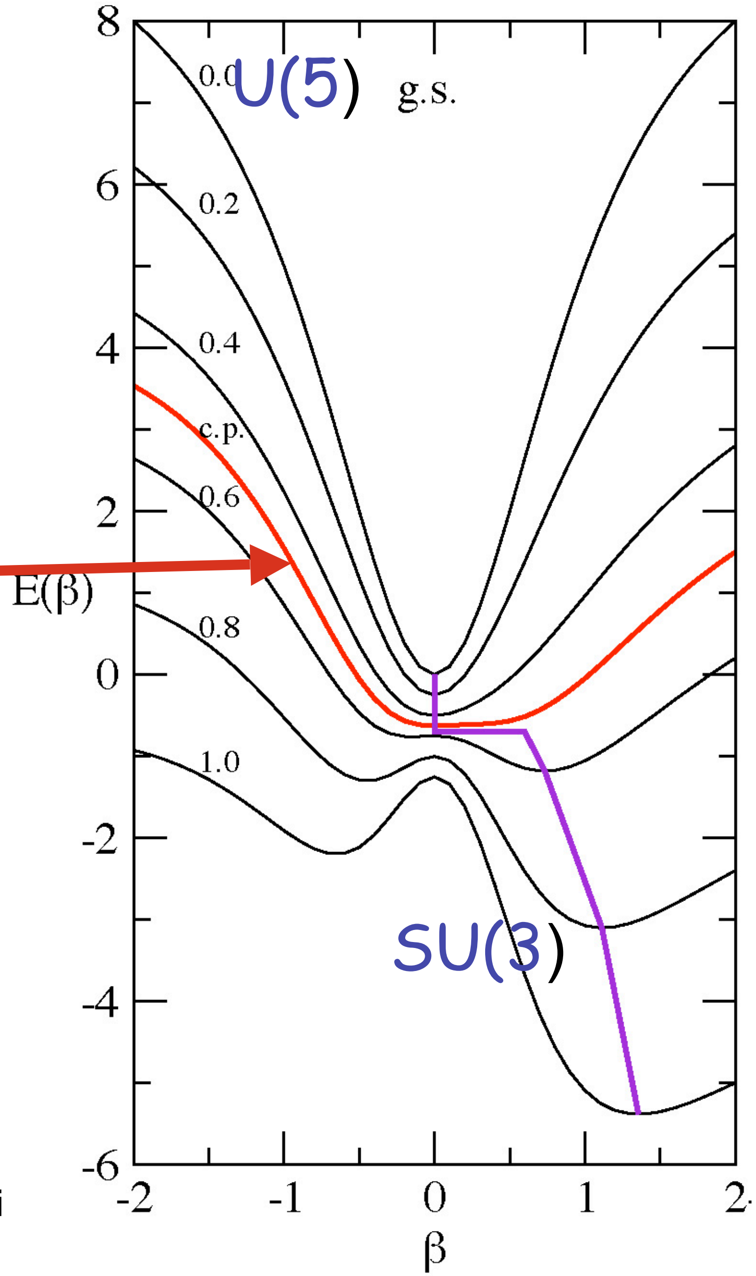
Energy surfaces E(β,γ=0)

U(5) to SU(3) transition
(varying the value of x)

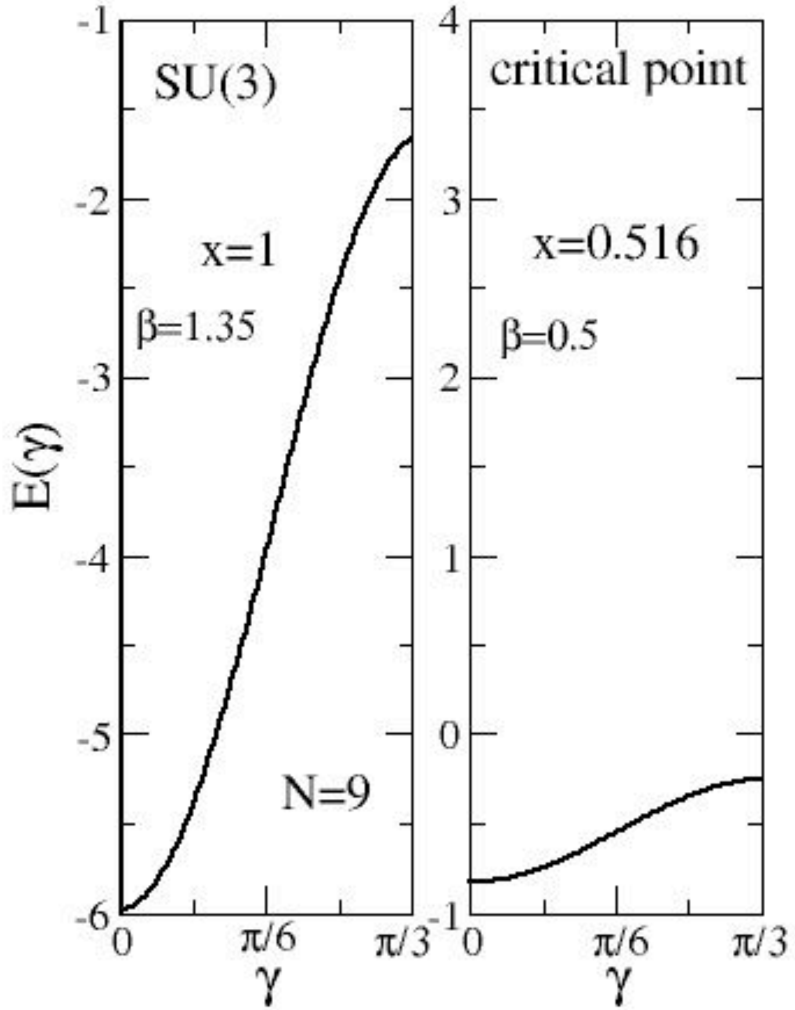
critical point

(first-order
phase transition)

x_{crit}=16N/(34N-27)
≈ 16/34 ≈ 0.5



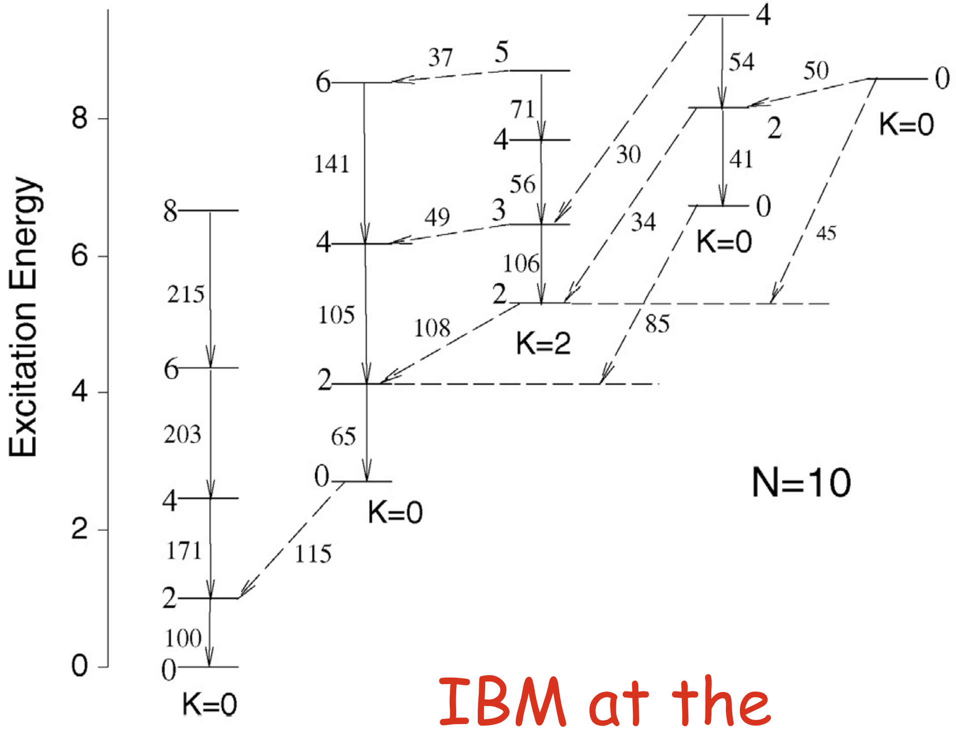
Energy surfaces $E(\beta=\beta_{min},\gamma)$



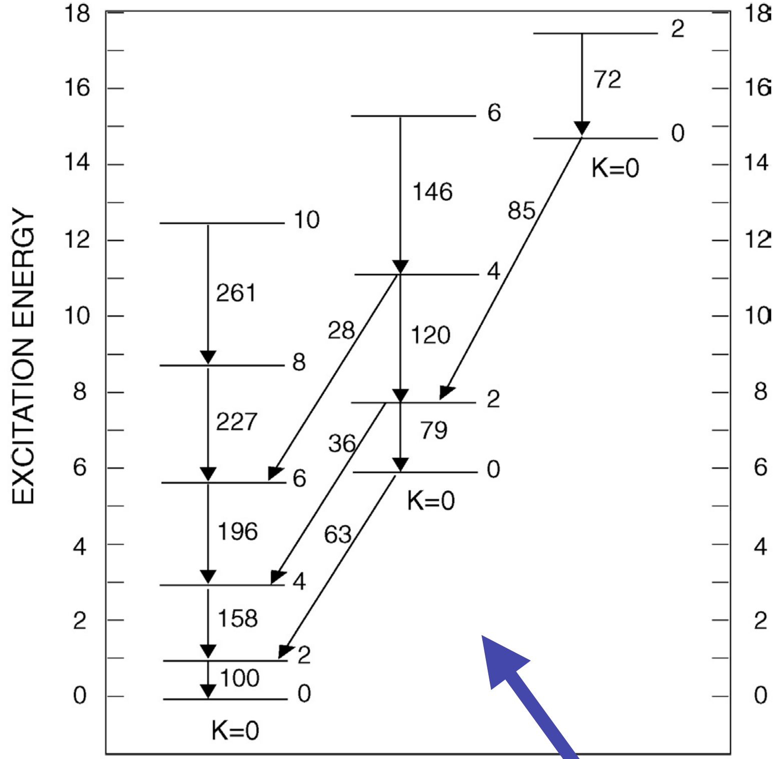
γ-dependence:

always a minimum for $\gamma=0$,
but steep dependence for x=1 (SU(3)) and rather smooth at the critical point

Even spectrum at the critical point



IBM at the critical point



Bohr Hamiltonian
X(5)

For large N
E(4+)/E(2+)≈2.43
E(0_2+)/E(2+)≈2.78

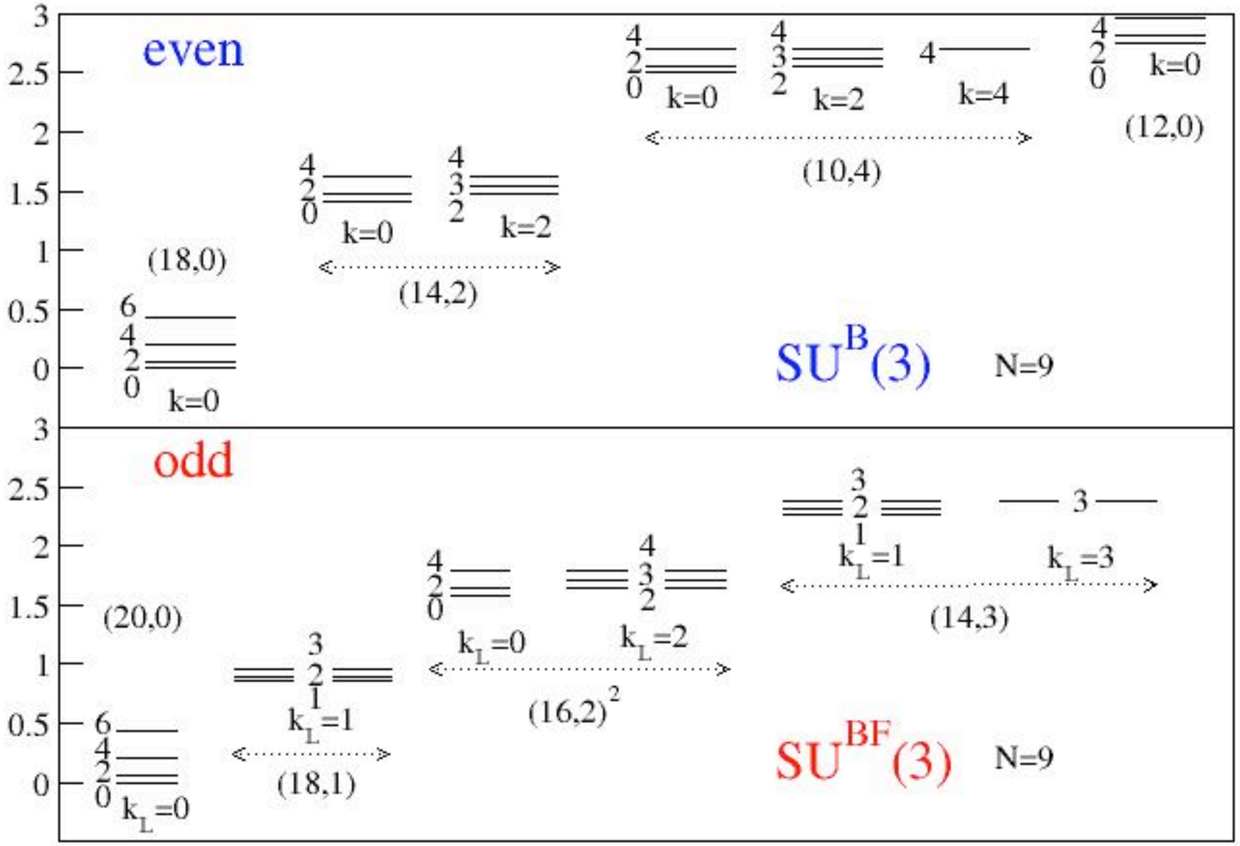
E(4+)/E(2+)=2.9
E(0_2+)/E(2+)=5.67

Following the previous lines, the odd system is described within the Interacting Boson Fermion Model (IBFM) with the model hamiltonian

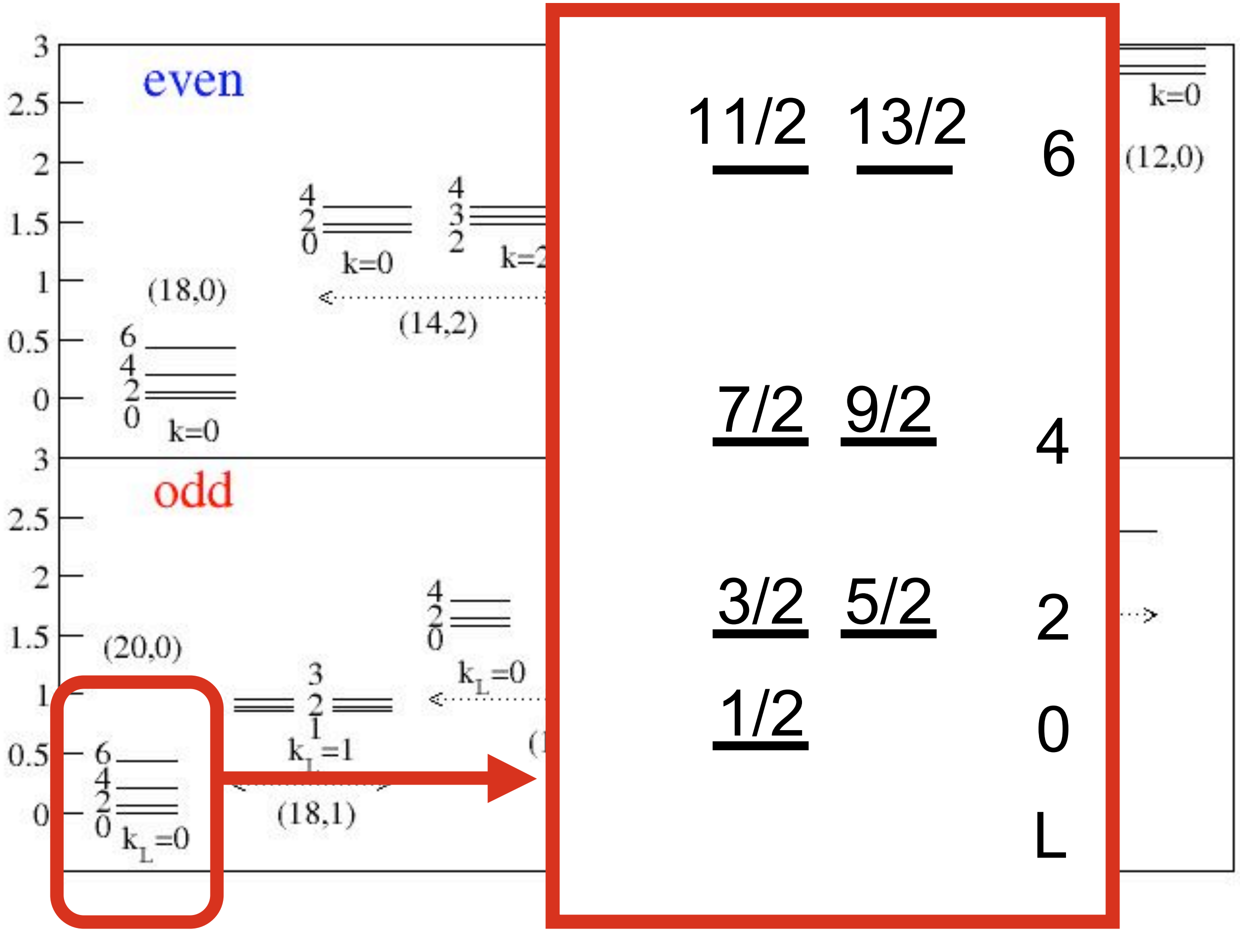
$$
\begin{aligned}
H^{BF} &= (1-x)(n_d + n_{3/2} + n_{5/2}) - \frac{x}{4N_B} Q_{BF}.Q_{BF} \\
&= (1-x)\ \mathcal{C}_1(U^{BF}5) \\
&- \frac{x}{8N_B}\left[\frac{3}{2}\ \mathcal{C}_2(SU^{BF}3) - \frac{3}{8}\ \mathcal{C}_2(O^{BF}3)\right]
\end{aligned}
$$

At the two extremes one obtains for x=0 the dynamical supersymmetry $U^{BF}(5)$ and for x=1 $SU^{BF}(3)$

At the SU(3) limit: even vs odd



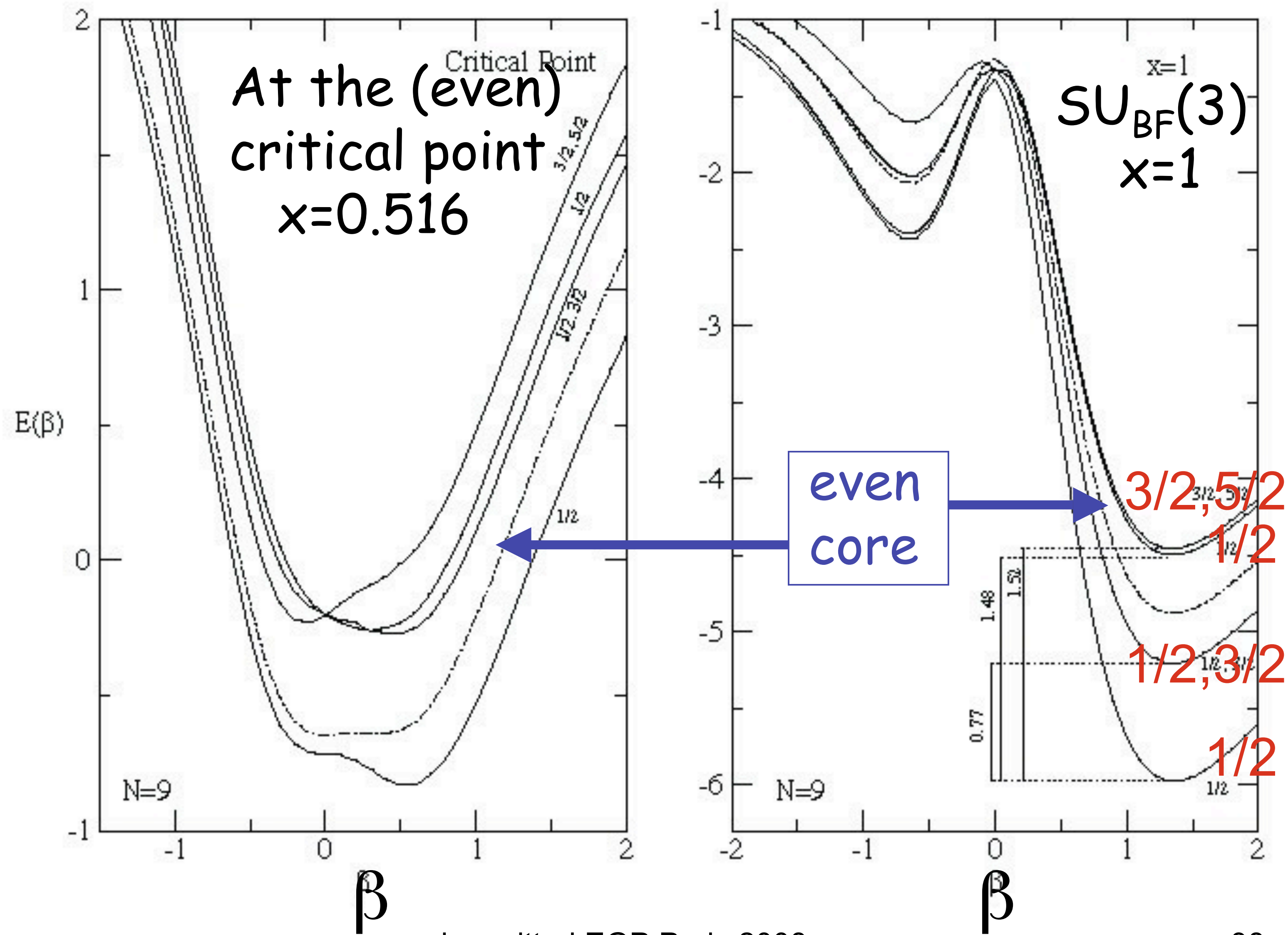
At the SU(3) limit: even vs odd



For generic value of x one can get a better understanding by resorting to the intrinsic states and the corresponding energy surfaces as a function of β and γ (obtained diagonalizing the boson-fermion interaction). One gets different Nilsson-like states.

In this case, for each state at the bottom of each energy surface one expect a rotational-like band (although not purely rigid rotational) in the laboratory

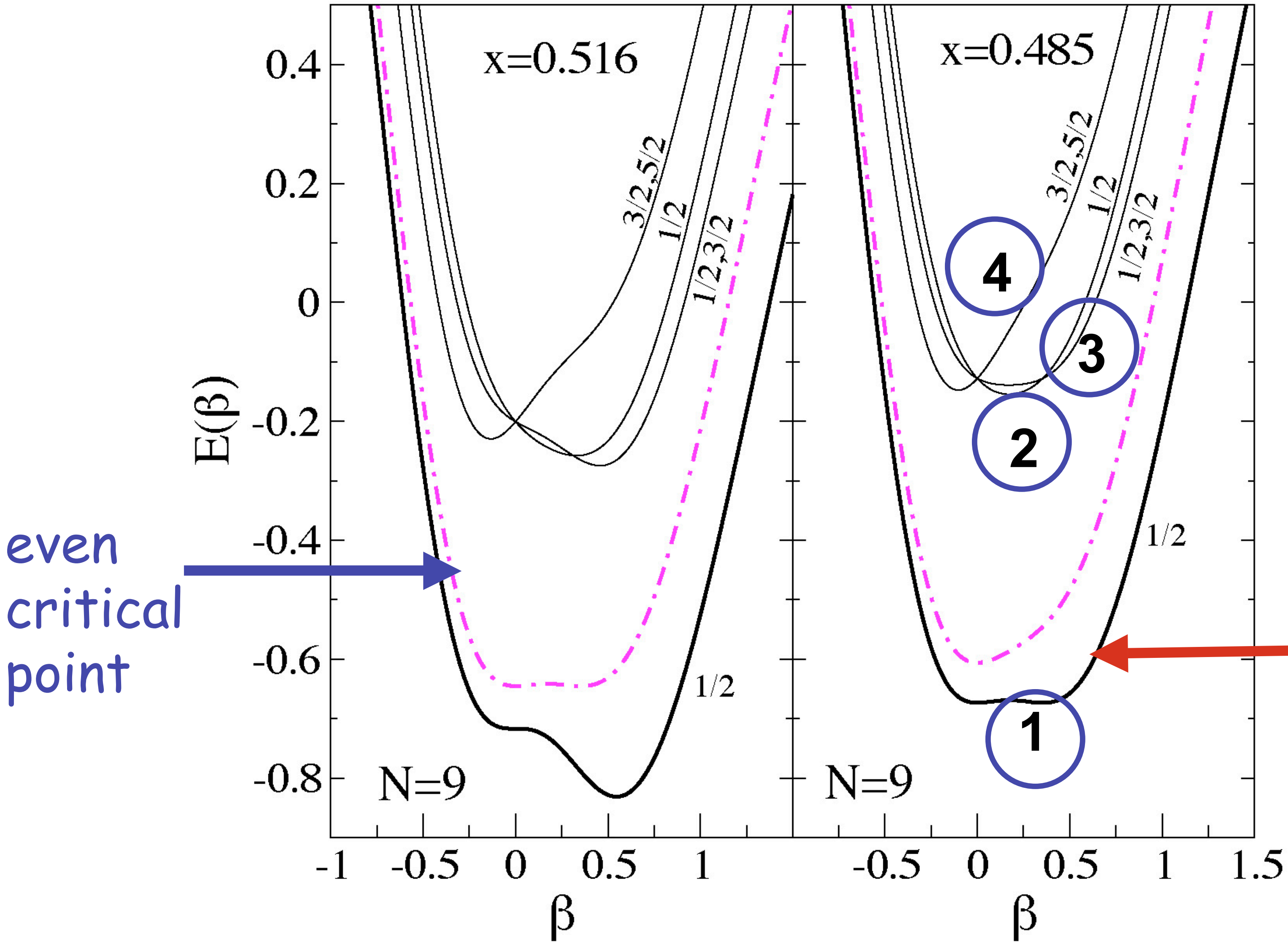
effect of odd particle on energy surfaces



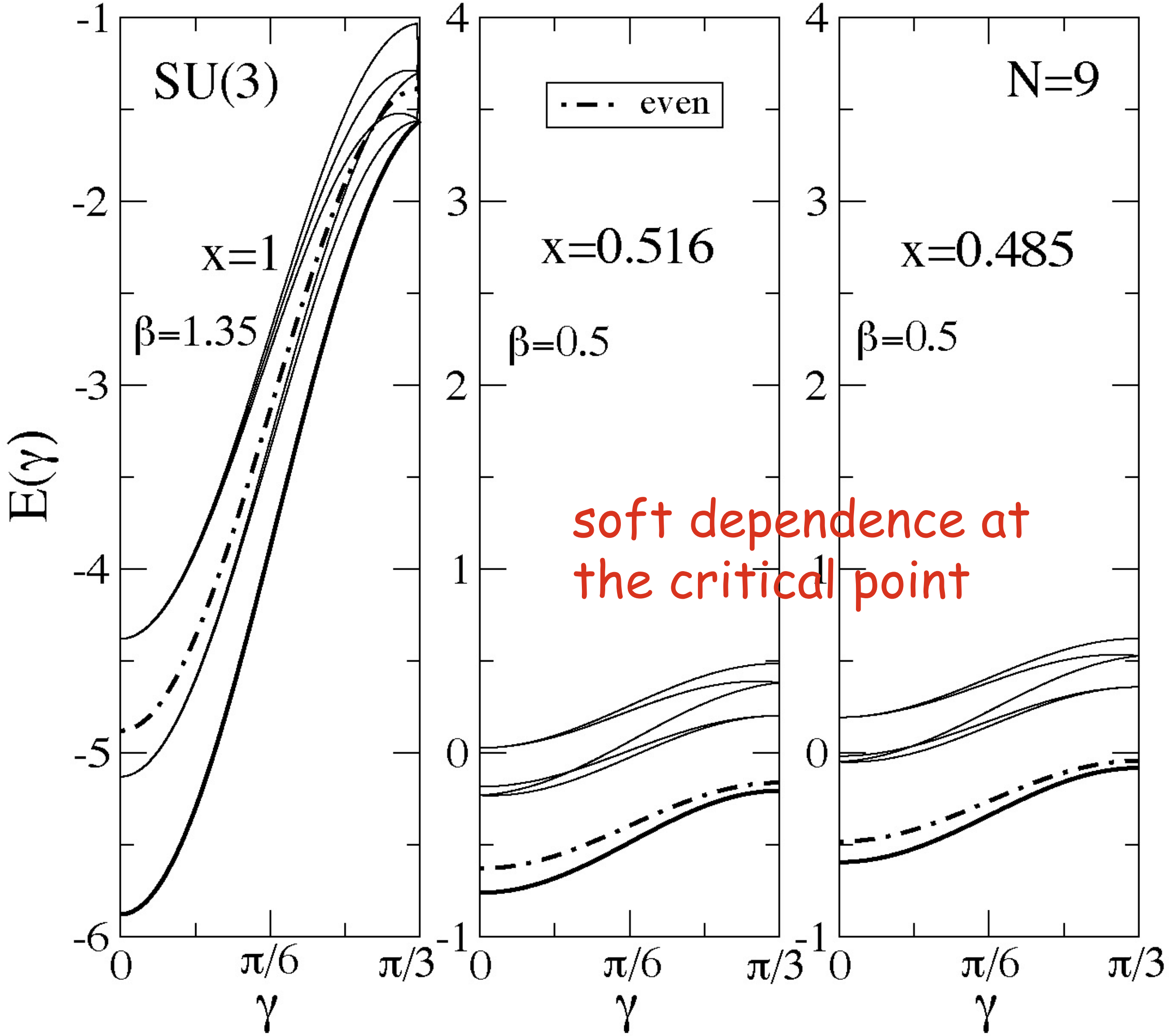
For well deformed systems all "effective" orbitals remain well deformed (all energy surfaces have the same minimum as the even part).

But around the critical point the situation is more "critical", since some orbitals are driving the system toward deformation and others towards sphericity. We have in fact a shift in the position of the critical point for the ground-band of the odd with respect to the ground-band of the even.

even/odd: shift in the position of the critical point

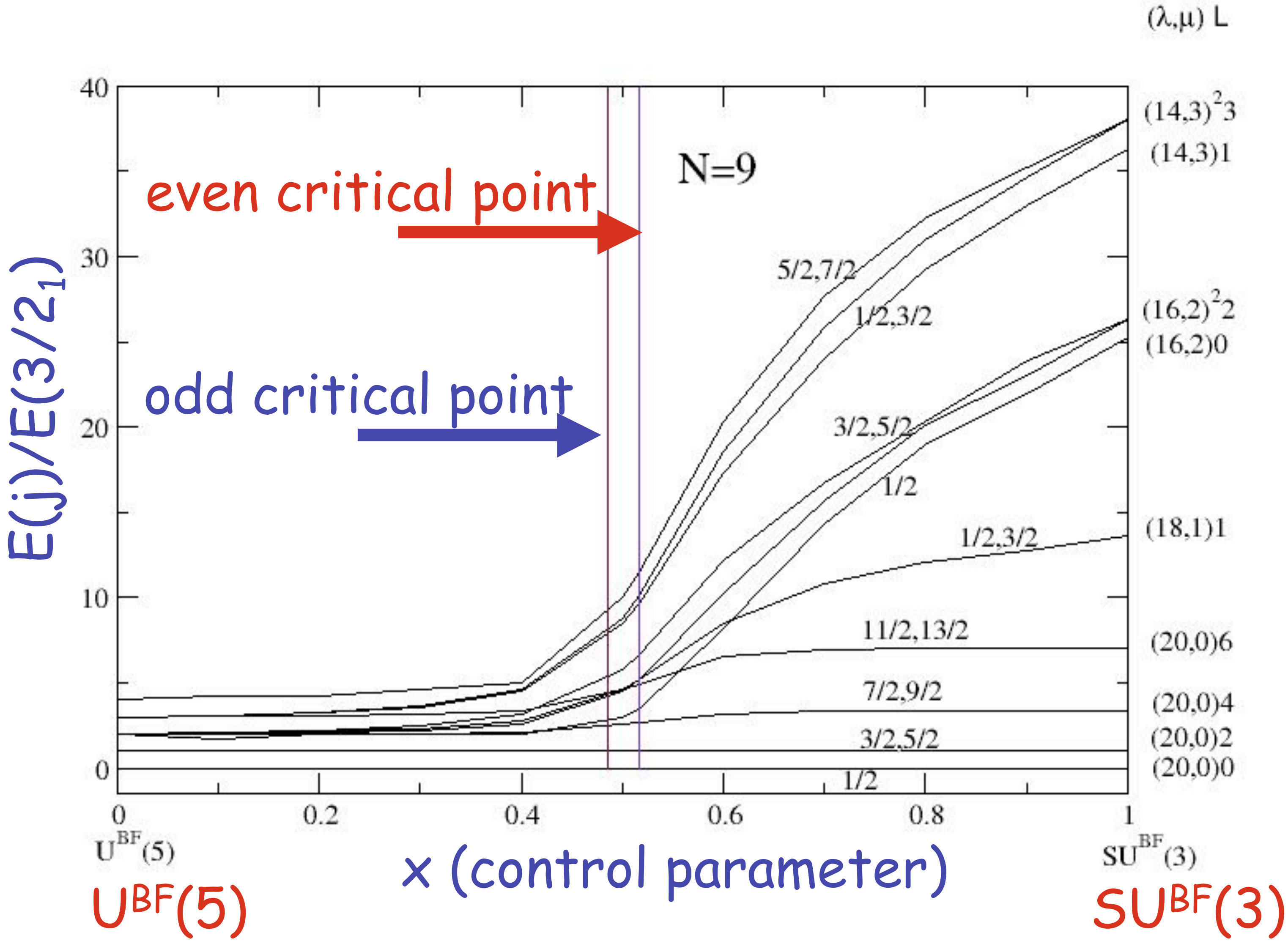


Energy surfaces: dependence on γ



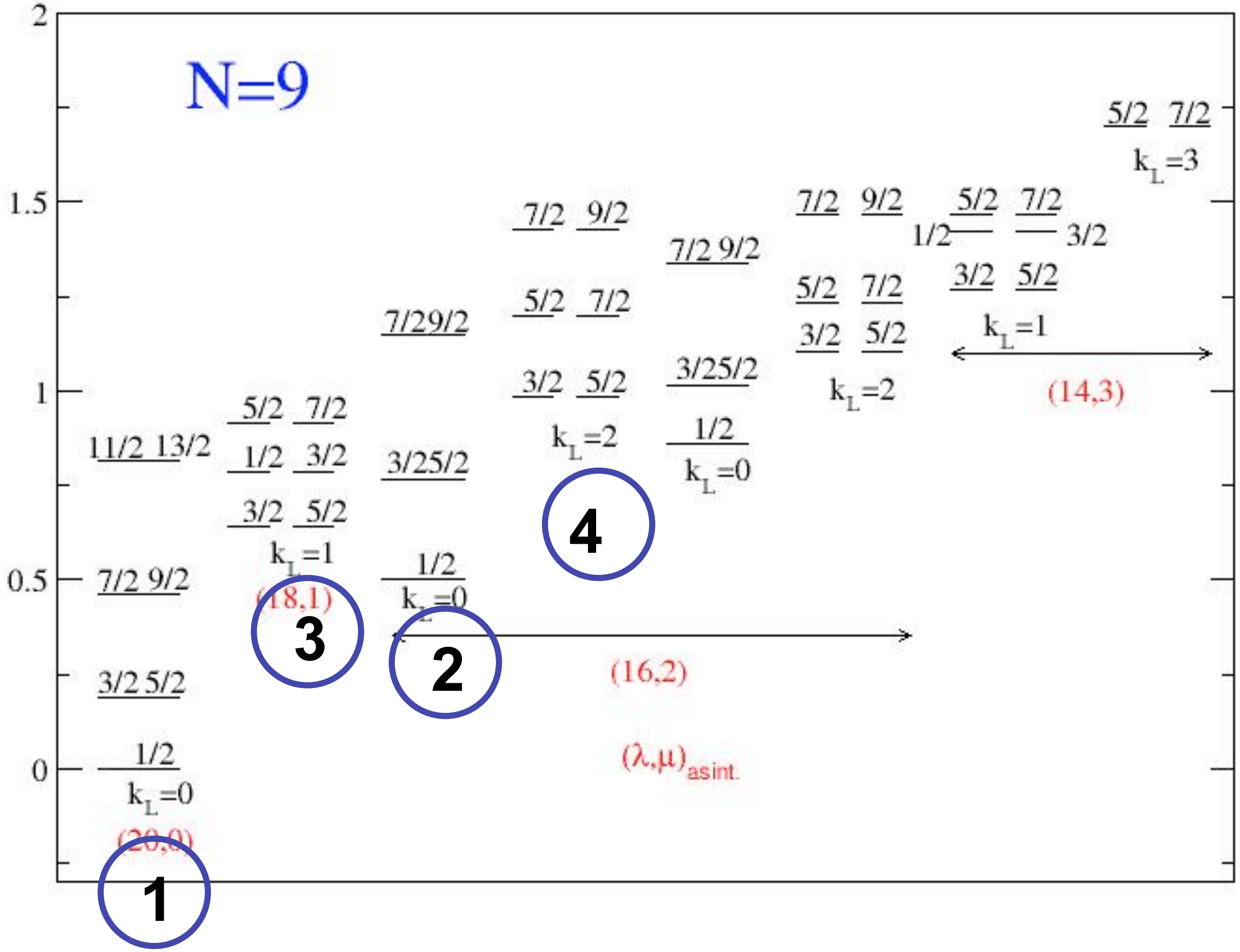
soft dependence at the critical point

Evolution of spectrum along the phase transition

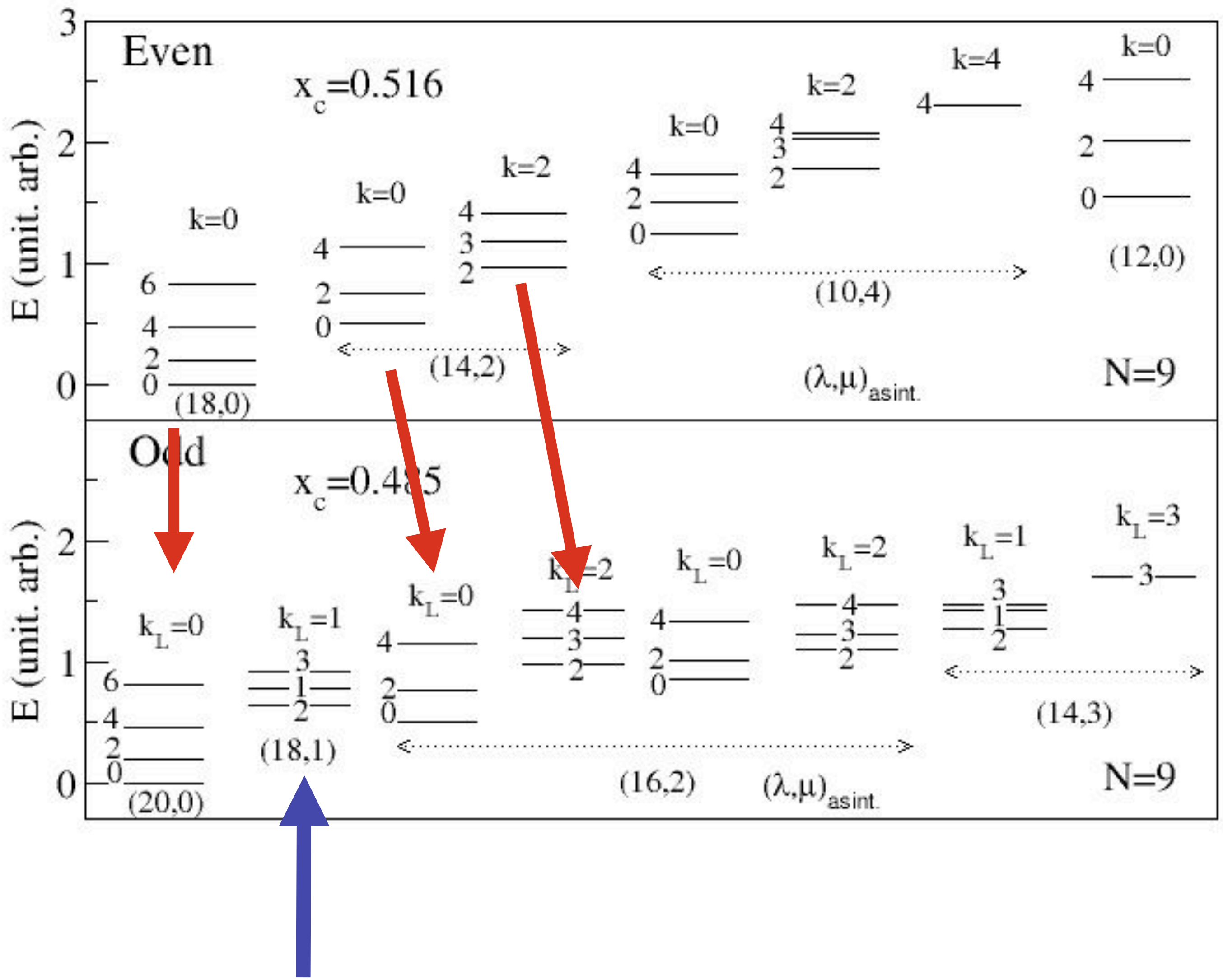


Spectrum at the critical point x=0.485

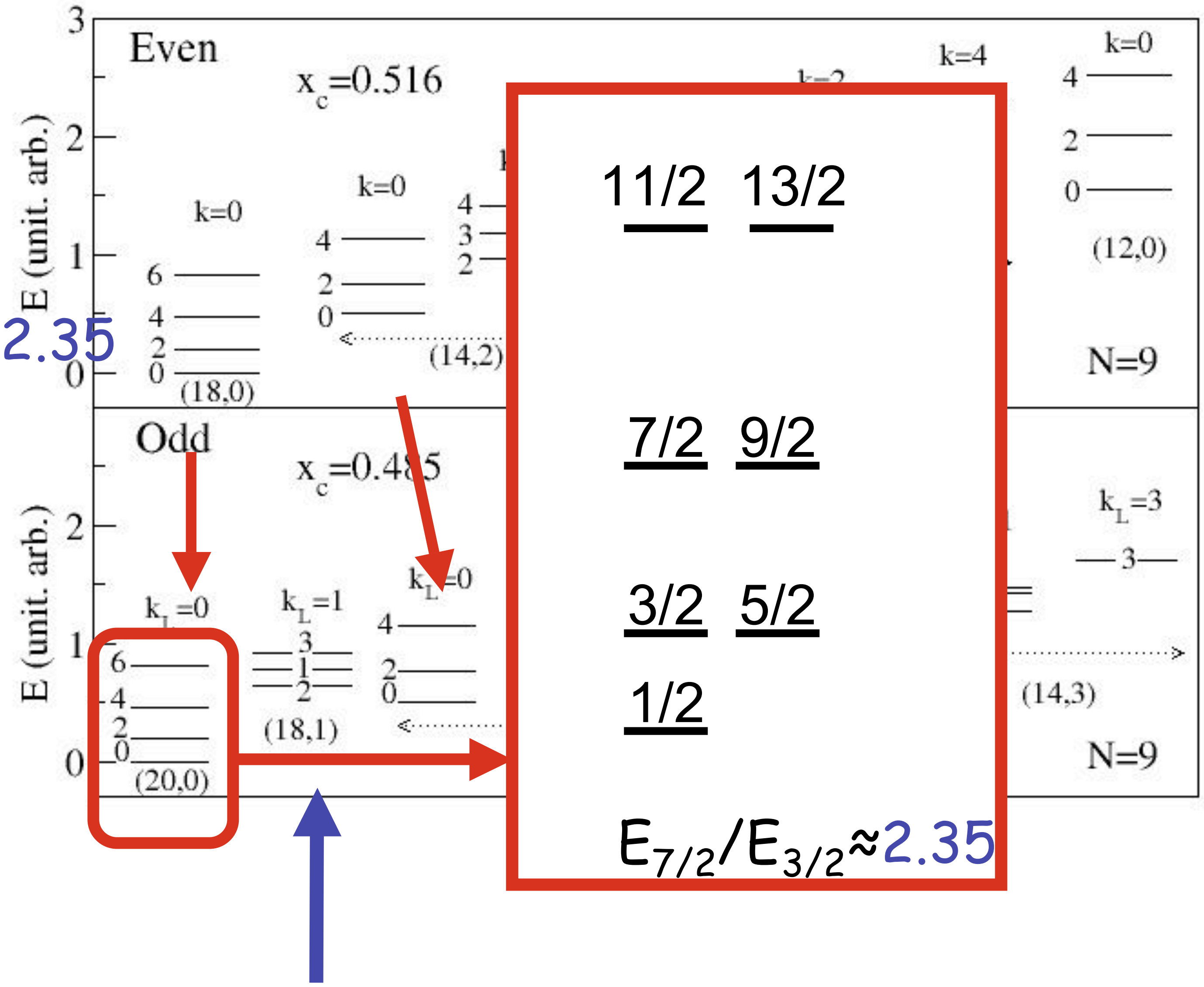
Each band corresponds to an intrinsic state



Comparison of spectra in even and odd nuclei



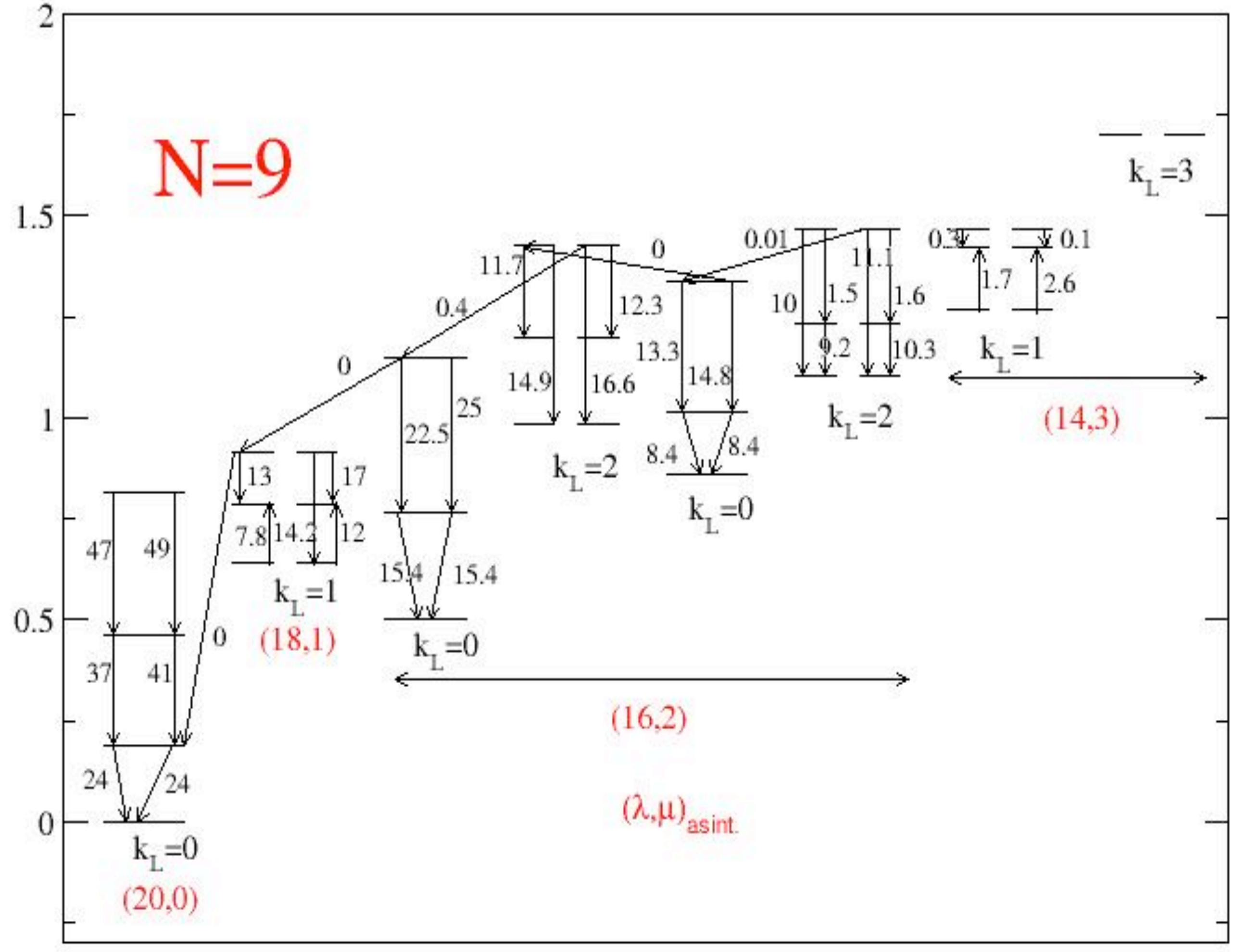
Comparison of spectra in even and odd nuclei



$E_4/E_2 \approx 2.35$

$E_{7/2}/E_{3/2} \approx 2.35$

B(Eλ) transition at the critical point x=0.485



Both spectrum and electromagnetic transitions in odd nuclei reflect the corresponding behaviours in the even core.

The value of the control parameter in the IBFM hamiltonian at the critical point is slightly modified with respect to the value in the even IBM hamiltonian. But the even and odd energy surfaces dispay at the corresponding critical point a rather similar behaviour.

Possible candidates for odd critical point symmetry close to X(5)

Nucleus		Single particle orbitals
Odd-proton	^{155}Tb (^{154}Gd)	$2d_{5/2}, 2d_{3/2}, 3s_{1/2}$
Odd-proton	^{103}Nb (^{104}Mo)	$2p_{1/2}, 2p_{3/2}, 1f_{5/2}$
Odd-neutron	^{105}Mo (^{104}Mo)	$2d_{5/2}, 2d_{3/2}, 3s_{1/2}$
Odd-proton	^{151}Pm (^{150}Nd)	$2d_{5/2}, 2d_{3/2}, 3s_{1/2}$
Odd-proton	^{153}Eu (^{152}Sm)	$2d_{5/2}, 2d_{3/2}, 3s_{1/2}$

Conclusions

Simple hamiltonian describing boson-fermion systems have been proposed along the so-called E(5) and X(5) critical points.

Particular boson-fermion symmetries are obtained with proper choice of the coupling hamiltonian and of the set of active fermion single-particle states (j=1/2,3/2 and 5/2).

At the critical points the spectra and the EM transitions display characteristic behaviours associated to the corresponding values in the even case. Odd nuclei provide therefore necessary signatures of the occurrence of the shape transition and critical point symmetries.